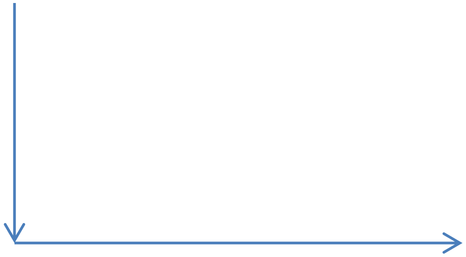
Image as a function

We can think of an **image** as a function, *f*, from R^2 to R:

- $f(x, y)$ gives the **intensity** at position (x, y)
  – Realistically, we expect the image only to be defined over a rectangle, with a finite range:
    – $f: [a,b] \times [c,d] \rightarrow [0,1]$

A color image is just three functions pasted together. We can write this as a "vector-valued" function:

$$f(x,y) = [r(x,y)\ g(x,y)\ b(x,y)]$$



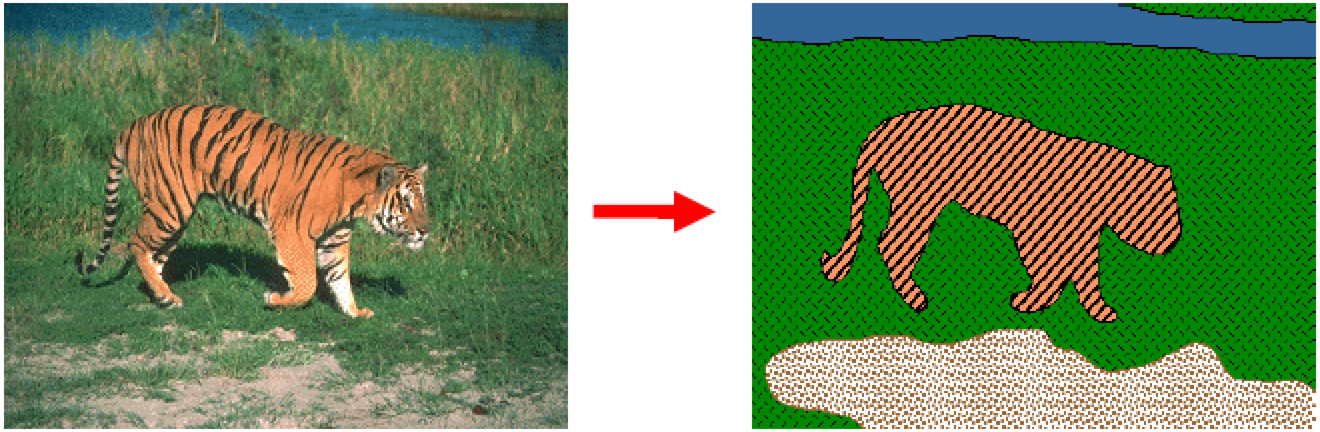
digital (discrete) images:

$j \rightarrow$

$i \downarrow$

62	79	23	119	120	105	4	0
10	10	9	62	12	78	34	0
10	58	197	46	46	0	0	48
176	135	5	188	191	68	0	49
2	1	1	29	26	37	0	77
0	89	144	147	187	102	62	208
255	252	0	166	123	62	0	31
166	63	127	17	1	0	99	30

To Extract Blobs



What are “blobs”?

- Regions of an image that are somehow coherent

Why?

- Object extraction, object removal, compositing, etc.
- …but are “blobs” objects?
- No, not in general

Simplest way to define blob coherence is as similarity in brightness or color:

Thresholding (Eşikleme)

Basic segmentation operation:

$$mask(x,y) = 1 \text{ if } im(x,y) > T$$
$$mask(x,y) = 0 \text{ if } im(x,y) < T$$

T is threshold

- User-defined
- Or automati

Same as histogram partitioning:

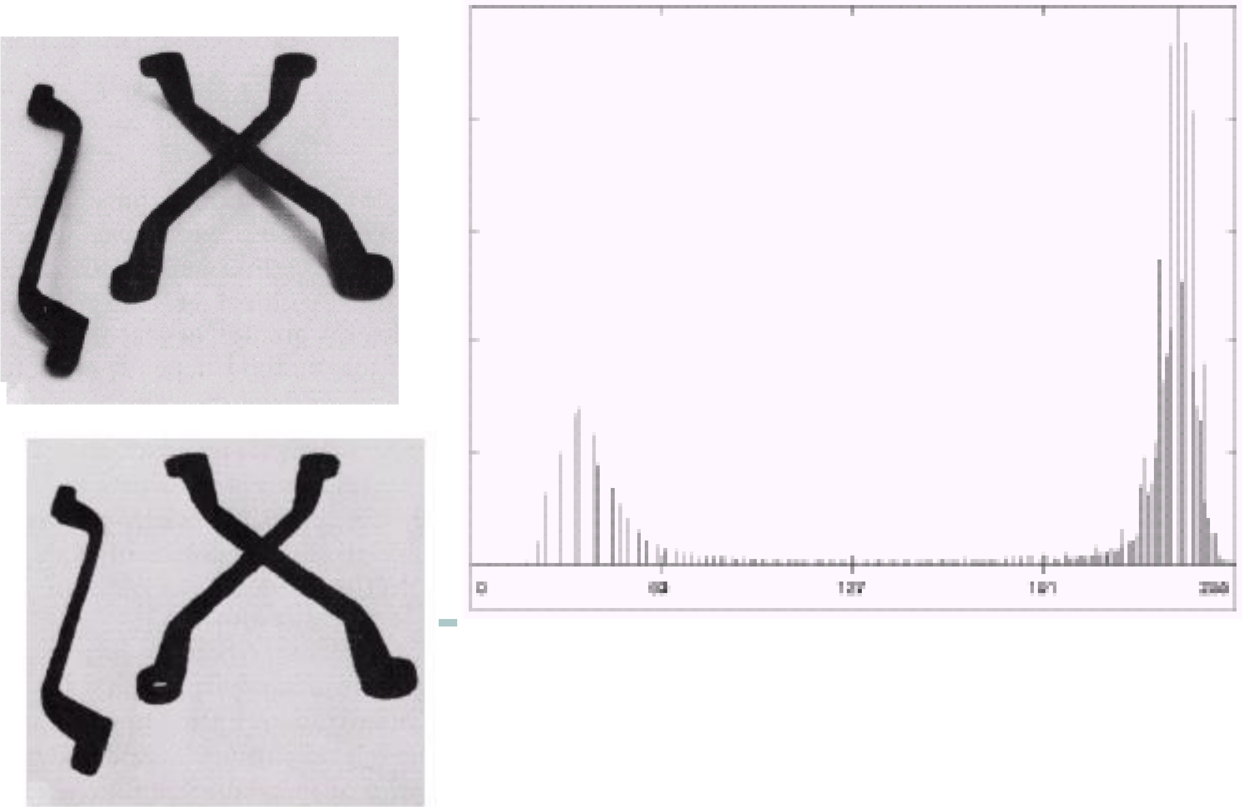


FIGURE 10.28 (a) Original image. (b) Image histogram. (c) Result of global thresholding with T midway between the maximum and minimum gray levels.

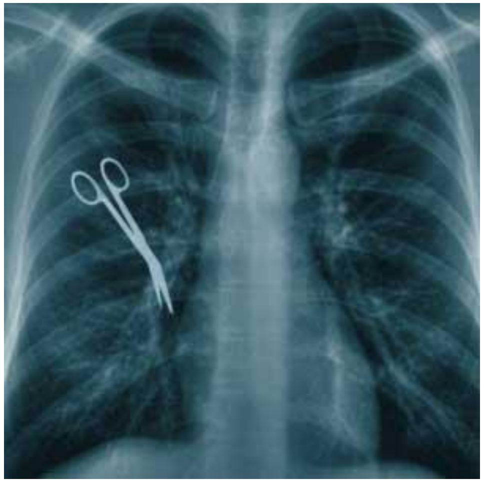
Binary Images

0 represents the background
1 represents the foreground

000100100001000
000111100001000
000100100001000


Document
Lorem ipsum dolor sit amet.
Consectetuer adipiscing elit. Cras non nunc nec
enim tristique tincidunt. Vestibulum quis tellus.
Duis nulla. Donec luctus urna. Sed tempus nibh
id massa. Vivamus placerat justo quis nibh. Ut
quis ante. Ut sollicitudin quam eu mi. Donec
molestie purus sit amet velit. Sed ac sem.
Aenean quis justo. Vestibulum ante ipsum primis
in faucibus orci luctus et ultrices posuere cubilia
Curae; Ut tincidunt.
Nulla facilisi. Aenean eros felis, blandit eu,
commodo sit amet, varius a, pede. Curabitur
augue felis, congue sed.

Binary images can be obtained from gray level images by thresholding



Operations After Thresholding

- Delete object pixels on boundary to better separate parts.
- Fill small holes
- Delete tiny objects

Salt-and-pepper noise

Binary mathematical morphology

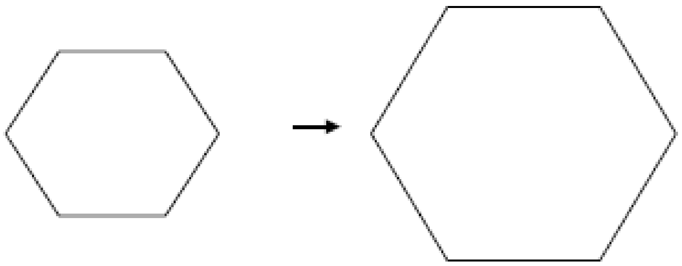
dilation **and** **erosion**

Dilation (Genleştirme)

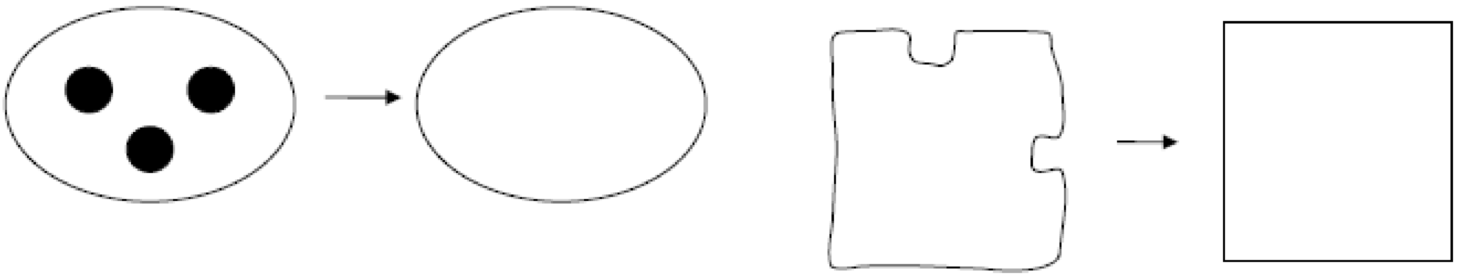
Dilation expands the connected sets of 1s of a binary image.

It can be used for

1. growing features



2. filling holes and gaps

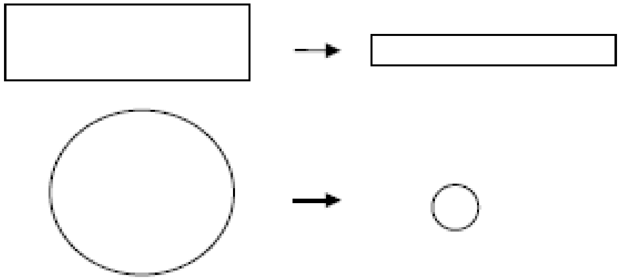


Erosion (Aşındırma)

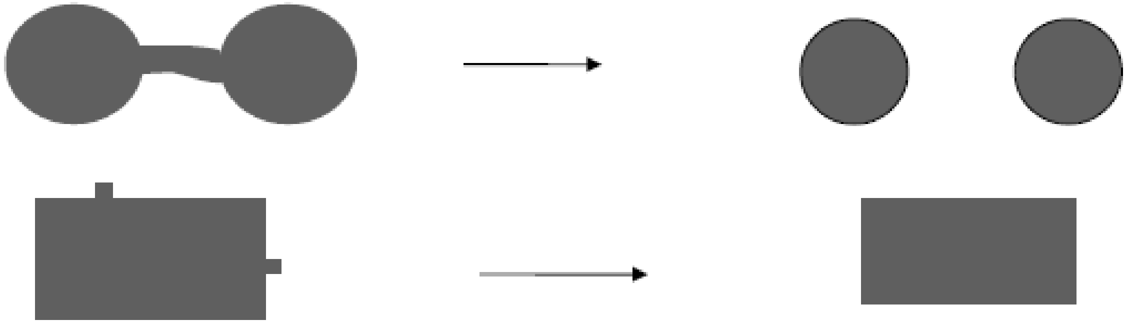
Erosion shrinks the connected sets of 1s of a binary image.

It can be used for

1. shrinking features



2. Removing bridges, branches and small protrusions



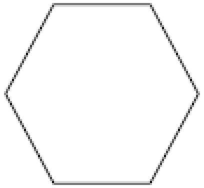
Structuring Element

A structuring element is a shape mask used in the basic morphological operations.

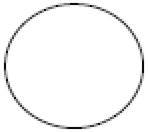
They can be any shape and size that is digitally representable, and each has an origin.

Yapısal Eleman

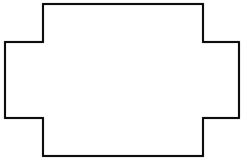
box



hexagon



disk



something

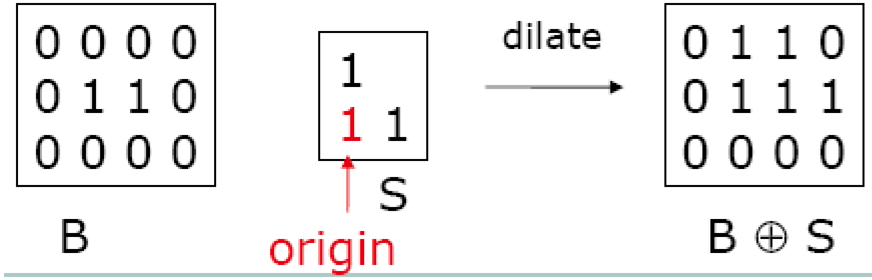
box(length,width)

disk(diameter)

Dilation on Binary Images

dilate(B,S) takes binary image B, places the origin of structuring element S over each 1-pixel, and ORs the structuring element S into the output image at the corresponding position.

a binary image B
a structuring element S



0	0	0	0
0	1	1	0
0	0	0	0

B

1	
1	1

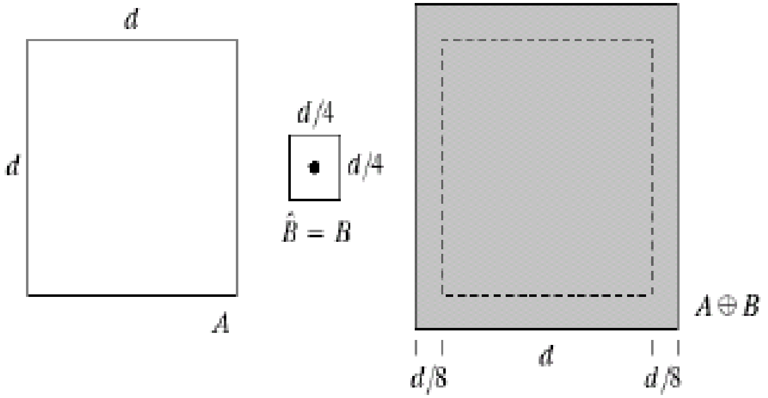
S

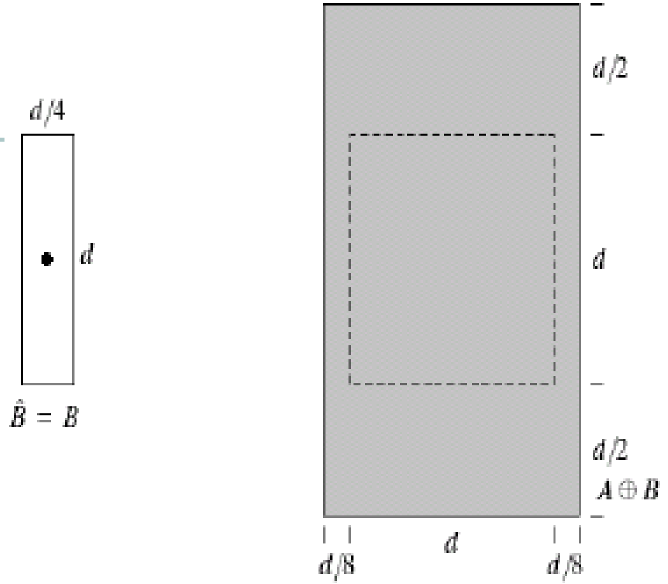
origin

dilate

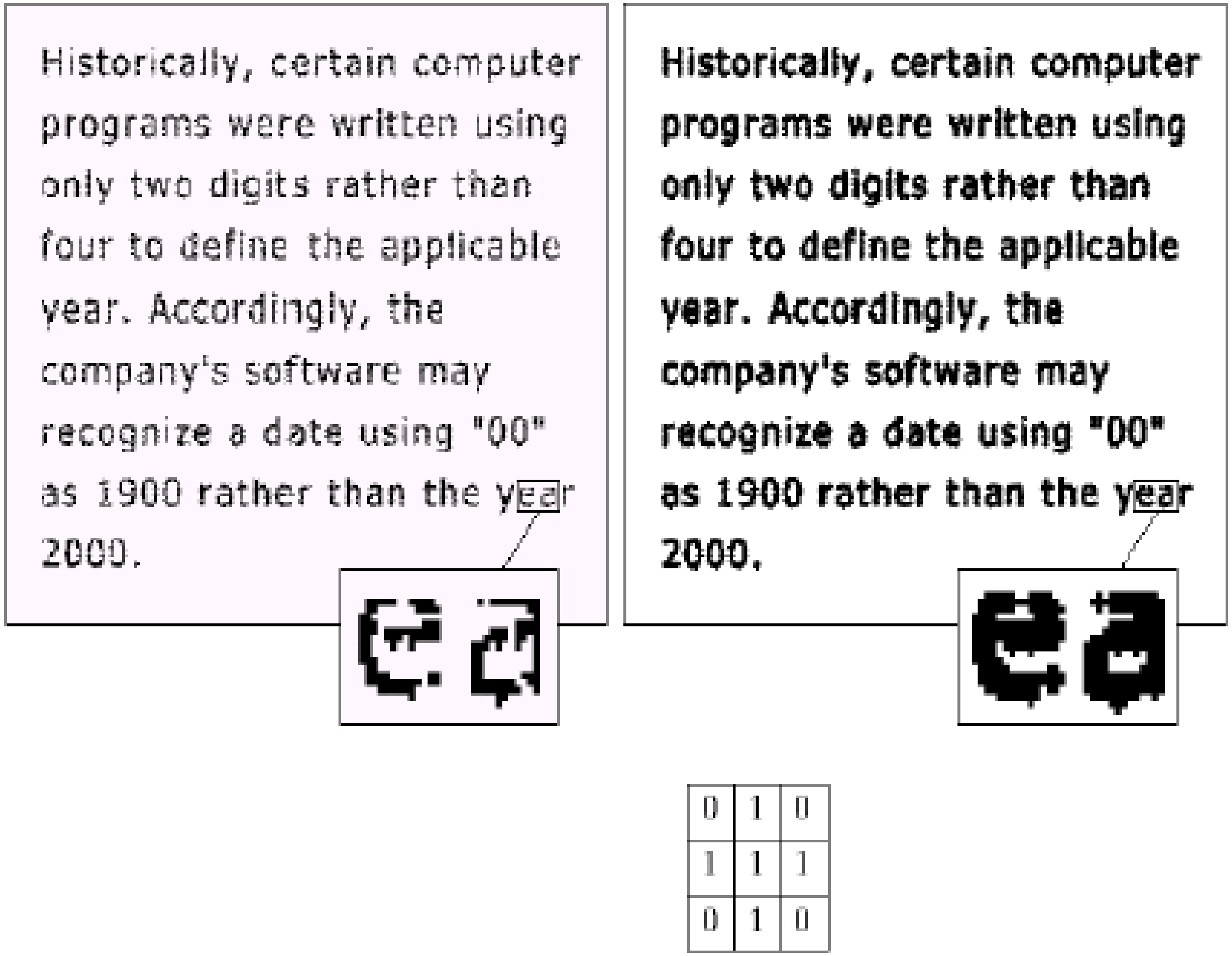
0	1	1	0
0	1	1	1
0	0	0	0

$B \oplus S$





Dilation on Document Image



Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.

Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.

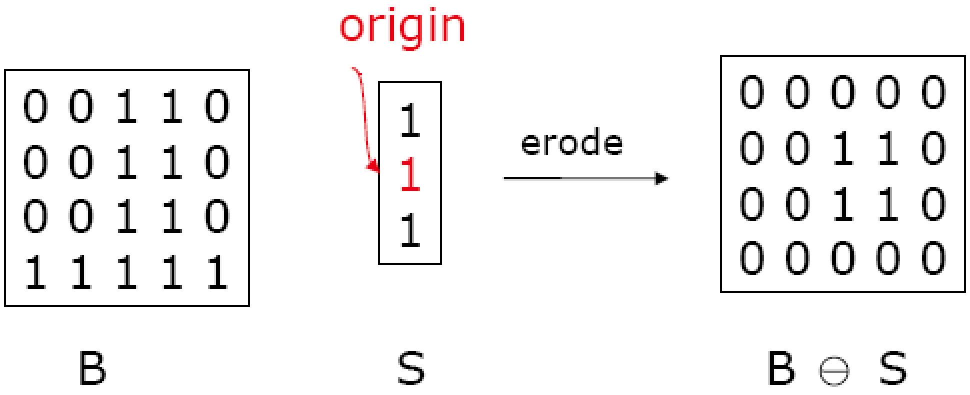
0	1	0
1	1	1
0	1	0

FIGURE 9.5
(a) Sample text of poor resolution with broken characters (magnified view).
(b) Structuring element.
(c) Dilation of (a) by (b). Broken segments were joined.

Erosion on Binary Images

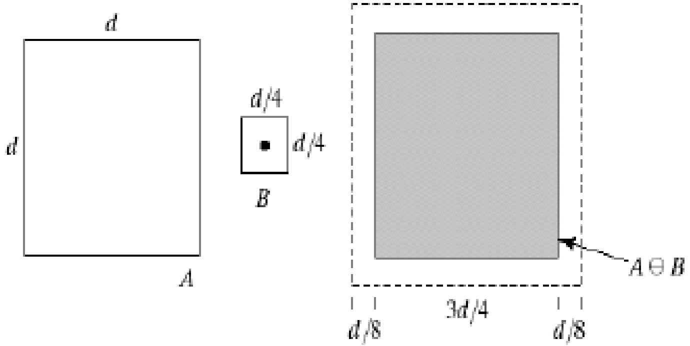
erode(B,S) takes a binary image B, places the origin of structuring element S over every pixel position, and ORs a binary 1 into that position of the output image only if every position of S (with a 1) covers a 1 in B.

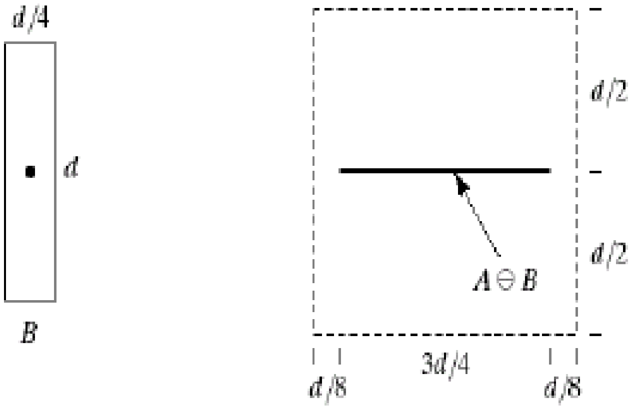
a binary image B
a structuring element S



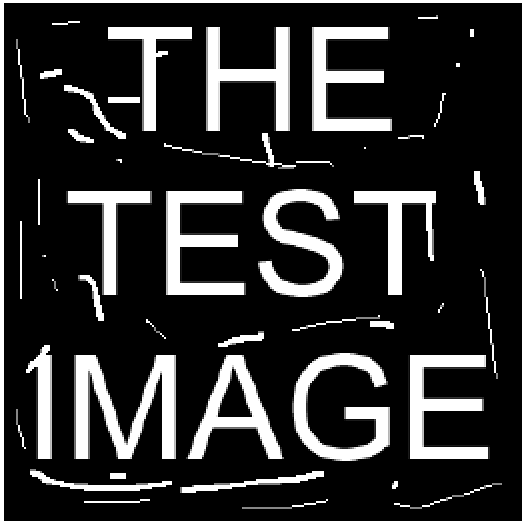
origin

$$\underset{B}{\begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}} \quad \underset{S}{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}} \quad \xrightarrow{\text{erode}} \quad \underset{B \ominus S}{\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}}$$

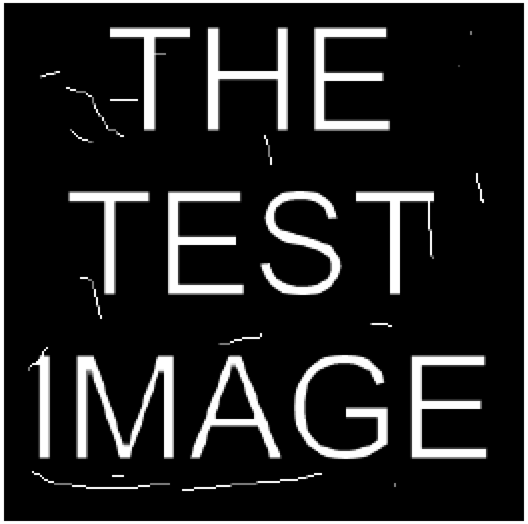




Erosion on Binary Images



Original image

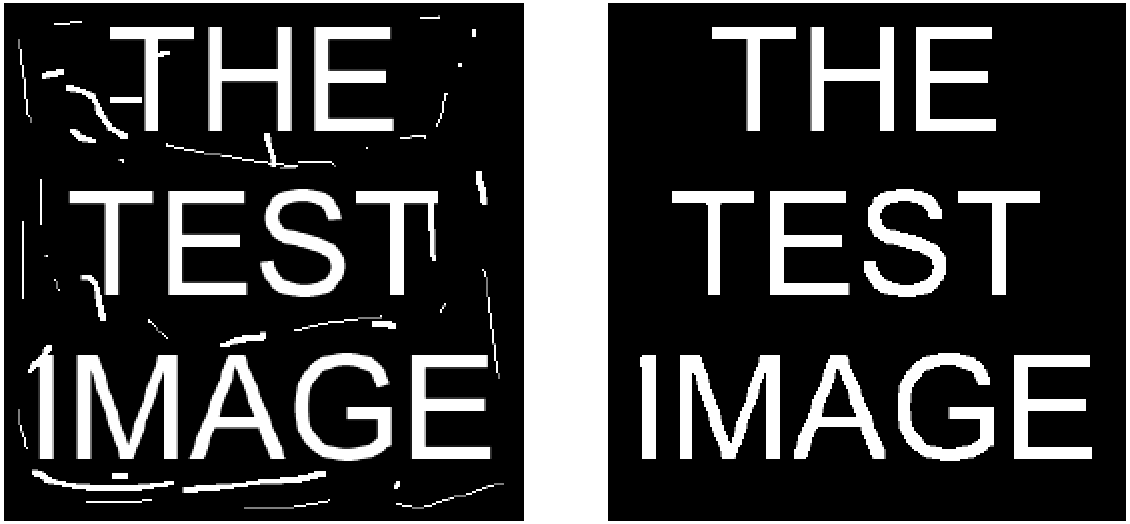


Eroded image

Eroded twice

Opening Operation

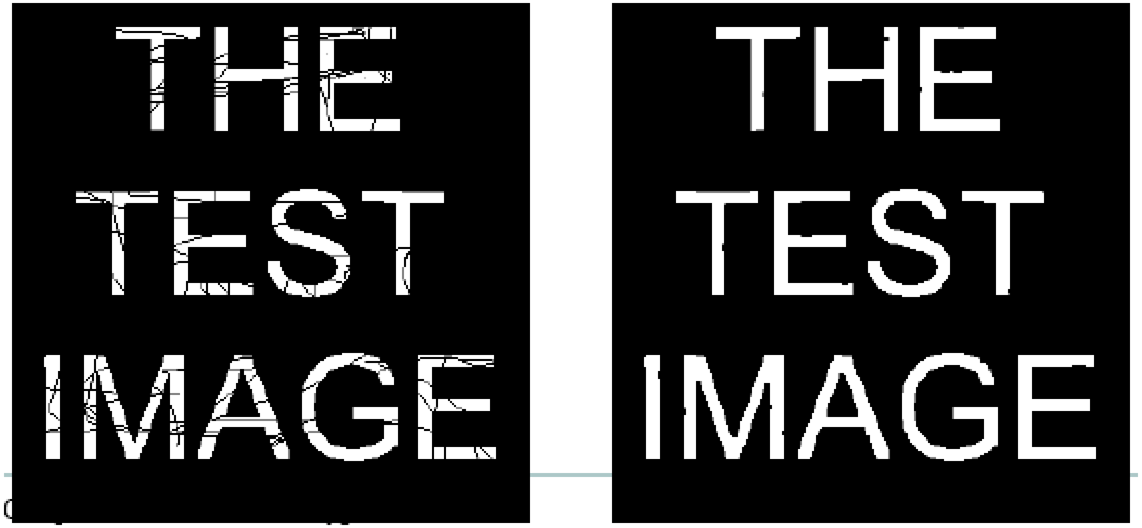
Opening is the compound operation of erosion followed by dilation (with the same structuring element)



OPENING: The original image eroded twice and dilated twice (opened). Most noise is removed

Closing Operation

Closing is the compound operation of dilation followed by erosion (with the same structuring element)



CLOSING: The original image dilated and then eroded. Most holes are filled.

1	1	1	1	1	1	1	
			1	1	1	1	
			1	1	1	1	
		1	1	1	1	1	
			1	1	1	1	
		1	1				

a) Binary image B

1	1	1
1	1	1
1	1	1

b) Structuring Element S

1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
	1	1	1	1	1	1	1
	1	1	1	1	1	1	1
	1	1	1	1	1	1	1
	1	1	1	1	1	1	1
	1	1	1	1			

c) Dilation $B \oplus S$

				1	1		
				1	1		
				1	1		

d) Erosion $B \ominus S$

	1	1	1	1	1	1	
		1	1	1	1	1	
		1	1	1	1	1	
		1	1	1	1	1	
		1	1	1	1	1	
		1	1				

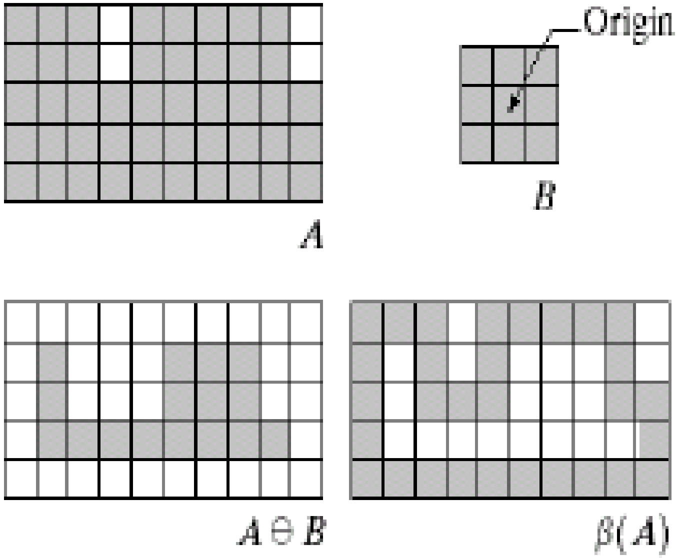
e) Closing $B \bullet S$

			1	1	1	1	
			1	1	1	1	
			1	1	1	1	
			1	1	1	1	
			1	1	1	1	

f) Opening $B \circ S$

Boundary Extraction

$\beta\ (A) = A - (A \Theta\ B)$ ***Difference between A and ERODE(A)***



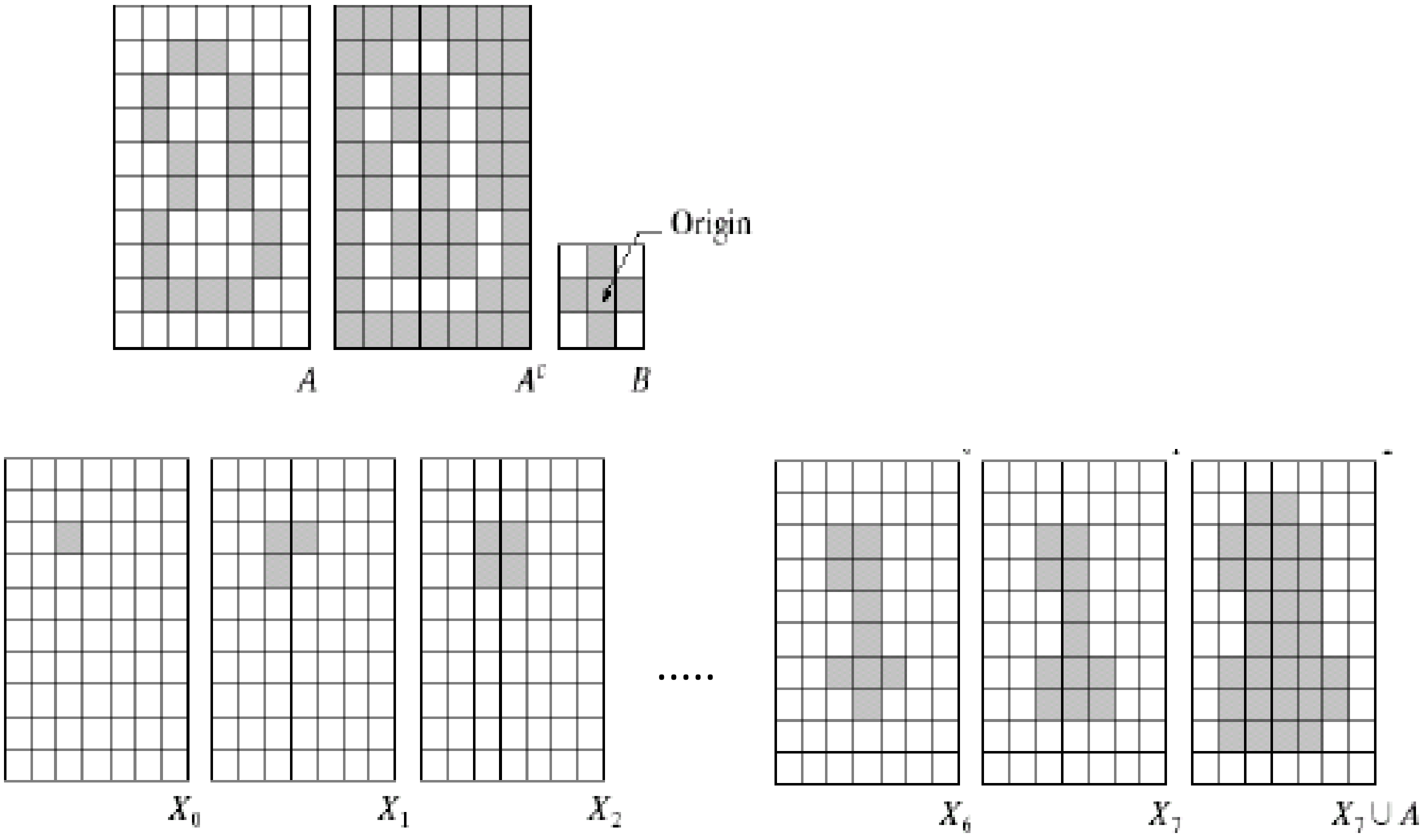
a b
c d

FIGURE 9.1
A. (b) Stru
element *B*. (c) *A*
eroded by *B*.
(d) Boundary, given
by the set
difference between
A and its erosion.

Region Filling

$$X_k = (X_{k-1} \oplus B) \cap A^c \quad k = 1,2,3....$$

FIGURE 9.15 Region filling. (a) Set *A*. (b) Complement of *A*. (c) Structuring element *B*. (d) Initial point inside the boundary. (e)–(h) Various steps of Eq. (9.5-2). (i) Final result [union of (a) and (h)].



Connected Component (Blobs)

Once you have a binary image, you can identify and then analyze each **connected set of pixels.**

$$X_k = (X_{k-1} \oplus B) \cap A \quad k = 1,2,3,....$$

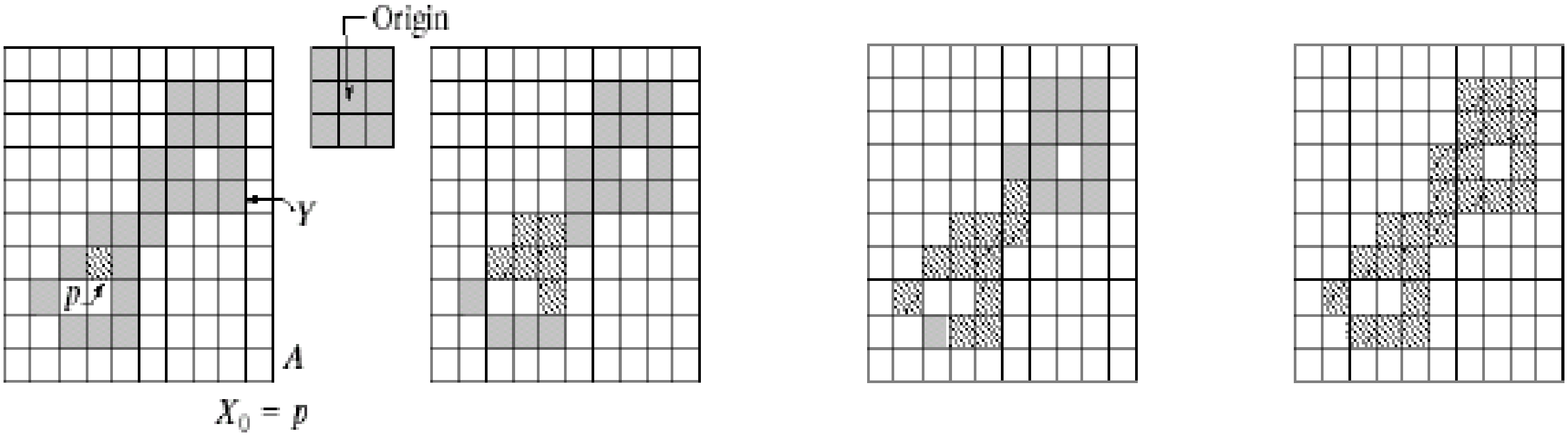


FIGURE 9.17 (a) Set A showing initial point p (all shaded points are valued 1, but are shown different from p to indicate that they have not yet been found by the algorithm). (b) Structuring element. (c) Result of first iterative step. (d) Result of second step. (e) Final result.