Private-Key Cryptography

- traditional **private/secret/single key** cryptography uses **one** key
- shared by both sender and receiver
- if this key is disclosed,
  - communications are compromised
- also is **symmetric**, parties are equal

- hence does not protect sender from receiver forging a message & claiming is sent by sender

Public-Key Cryptography

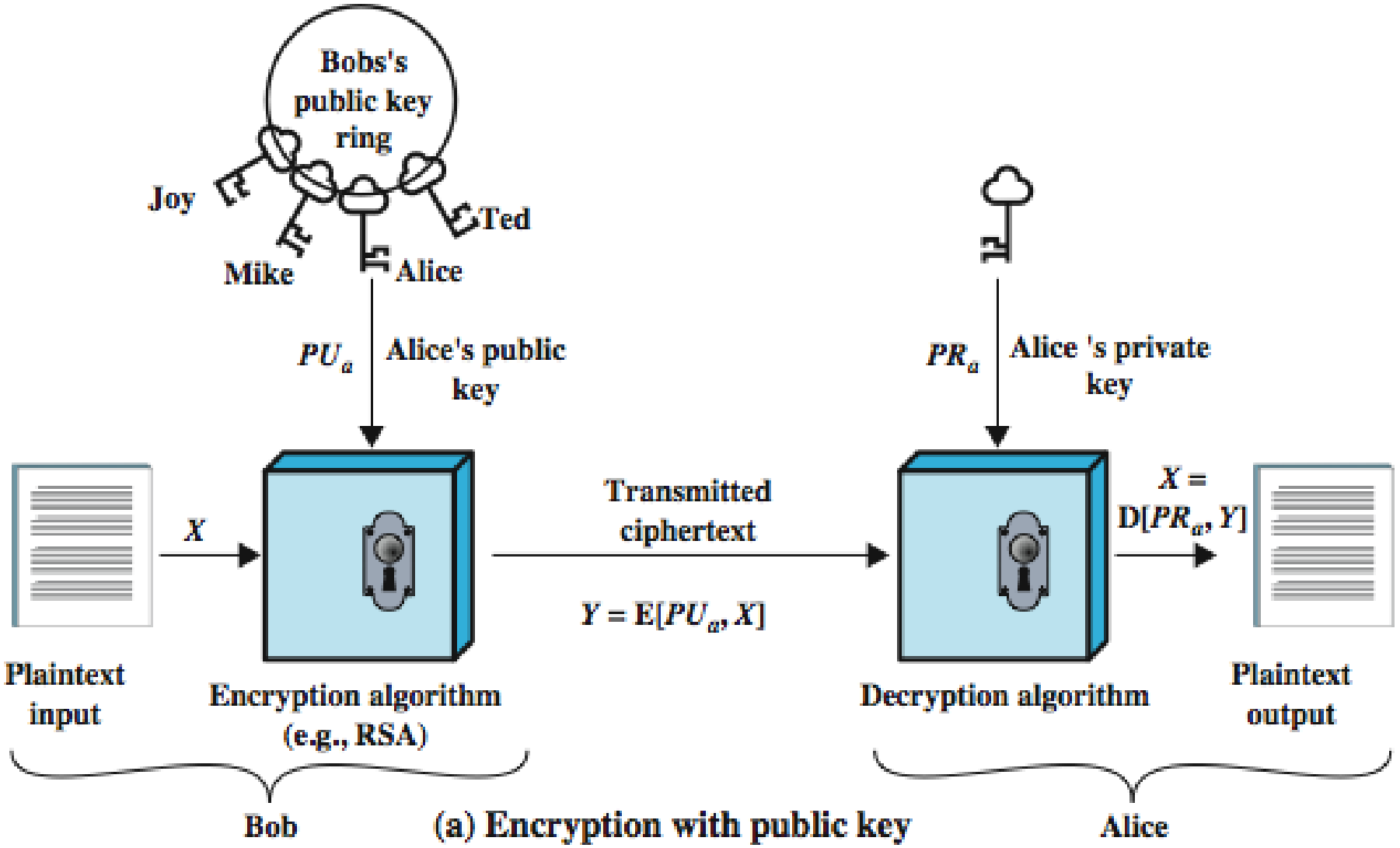
- probably most significant advance in the 3000 year history of cryptography
- uses **two** keys – a public & a private key
- **asymmetric** since parties are **not** equal
- uses clever application of number theory
- anyone knowing the public key **can** encrypt messages or verify signatures, but **cannot** decrypt messages or create signatures

Why Public-Key Cryptography?

- developed to address two key issues of private-key crypto:
  - **key distribution** – how to have secure communications in general without having to trust a KDC (key distribution center) with your key
  - **digital signatures** – how to verify a message comes intact from the claimed sender
- public invention due to Diffie & Hellman 1976

Public-Key Cryptography

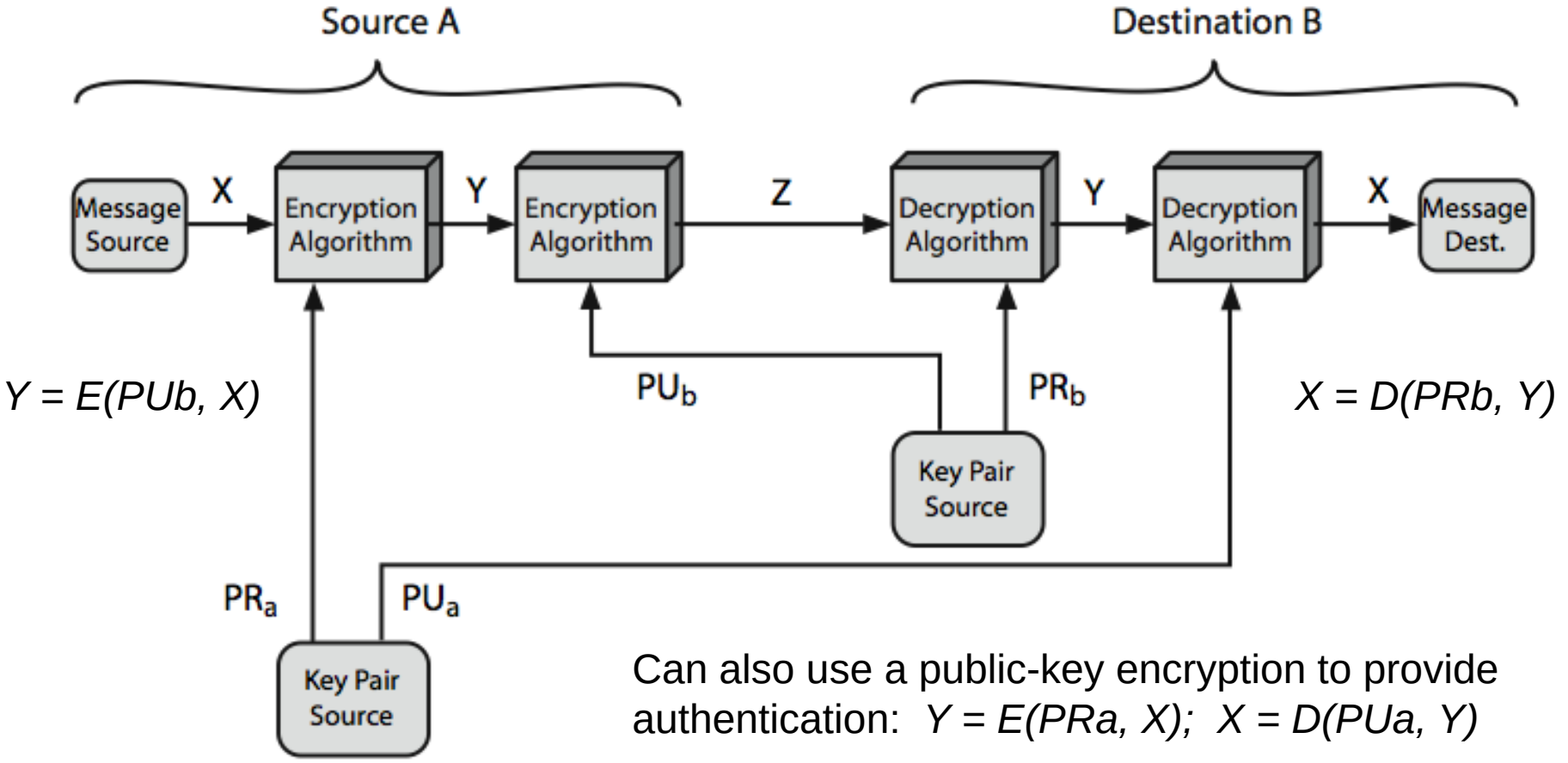
- **public-key/two-key/asymmetric** cryptography involves the use of **two** keys:
  - a **public-key**, which may be known by anybody, and can be used to **encrypt messages**, and **verify signatures**
  - a **private-key**, known only to the recipient, used to **decrypt messages**, and **sign** (create) **signatures**
- **infeasible to determine private key from public**
- is **asymmetric** because
  - those who encrypt messages or verify signatures **cannot** decrypt messages or create signatures



(a) Encryption with public key

Conventional Encryption	Public-Key Encryption
Needed to Work: 1. The same algorithm with the same key is used for encryption and decryption. 2. The sender and receiver must share the algorithm and the key. *Needed for Security:* 1. The key must be kept secret. 2. It must be impossible or at least impractical to decipher a message if no other information is available. 3. Knowledge of the algorithm plus samples of ciphertext must be insufficient to determine the key.	*Needed to Work:* 1. One algorithm is used for encryption and decryption with a pair of keys, one for encryption and one for decryption. 2. The sender and receiver must each have one of the matched pair of keys (not the same one). *Needed for Security:* 1. One of the two keys must be kept secret. 2. It must be impossible or at least impractical to decipher a message if no other information is available. 3. Knowledge of the algorithm plus one of the keys plus samples of ciphertext must be insufficient to determine the other key.

Public-Key Cryptosystems



Can also use a public-key encryption to provide authentication: *Y = E(PRa, X); X = D(PUa, Y)*

To provide both authentication and confidentiality, have a double use of the public-key scheme (as shown here): *Z = E(PUb, E(PRa, X)) X = D(PUa, D(PRb, Z))*

- Public-Key algorithms rely on two keys where:
  - it is computationally infeasible to find decryption key knowing only algorithm & encryption key
  - it is computationally easy to en/decrypt messages when the relevant (en/decrypt) key is known

1.It is computationally easy for a party B to generate a pair (public key *PUb,* private key PRb*).*

2.It is computationally easy for a sender A, knowing the public key and the message to be encrypted, *M*, to generate the corresponding ciphertext: C = E(PUb, M)

3.It is computationally easy for the receiver B to decrypt the resulting ciphertext using the private key to recover the original message:
M = D(PRb, C) = D[PRb, E(PUb, M)

4.It is computationally infeasible for an adversary, knowing the public key, Pb, to determine the private key, PRb

5.It is computationally infeasible for an adversary, knowing the public key, Pb, and a ciphertext, C, to recover the original message, M.

Public-Key Requirements

- need a trapdoor one-way function
- one-way function has
  - $Y = f(X)$ easy
  - $X = f^{-1}(Y)$ infeasible
- a trap-door one-way function has
  - $Y = f_k(X)$ easy, if k and X are known
  - $X = f_k^{-1}(Y)$ easy, if k and Y are known
  - $X = f_k^{-1}(Y)$ infeasible, if Y known but k not known
- a practical public-key scheme depends on a suitable trap-door one-way function

Security of Public Key Schemes

- like private key schemes brute force **exhaustive search** attack is always theoretically possible
- but keys used are too large (>512bits)
- security relies on a **large enough** difference in difficulty between **easy** (en/decrypt) and **hard** (cryptanalyse) problems
- more generally the **hard** problem is known, but is made hard enough to be impractical to break
- requires the use of **very large numbers**
- hence is **slow** compared to private key schemes

RSA

- by Rivest, Shamir & Adleman of MIT in 1977
- best known & widely used public-key scheme
- based on exponentiation in a finite (Galois) field over integers modulo a prime
  - nb. exponentiation takes $O((\log n)^3)$ operations (easy)
- uses large integers (eg. 1024 bits)
- security due to cost of factoring large numbers
  - nb. factorization takes $O(e^{\log n \log \log n})$ operations (hard)

RSA En/decryption

- to encrypt a message M the sender:
  - obtains **public key** of recipient $PU=\{e,n\}$
  - computes: $C = M^e \bmod n$, where $0 \leq M < n$
- to decrypt the ciphertext C the owner:
  - uses their private key $PR=\{d,n\}$
  - computes: $M = C^d \bmod n$
- plaintext is encrypted in blocks, with each block having a binary value M less than some number n

RSA Key Setup

- each user generates a public/private key pair by:
- selecting two large primes at random: p, q
- computing their system modulus $n=p.q$
  - $\phi(n)=(p-1)(q-1)$
- selecting at random the encryption key e
  - where $1<e<\phi(n), \gcd(e,\phi(n))=1$
- solve following equation to find decryption key d
  - $e.d=1 \bmod \phi(n)$ and $0 \leq d \leq n$
- publish their public encryption key: PU={e,n}
- keep secret private decryption key: PR={d,n}

Why RSA Works

- because of Euler's Theorem:
  - $a^{\phi(n)} \bmod n = 1$ where $\gcd(a,n)=1$
- in RSA:
  - $n=p.q$
  - $\phi(n)=(p-1)(q-1)$
  - carefully chose e & d to be inverses $\bmod \phi(n)$
  - hence $e.d=1+k.\phi(n)$ for some k

- hence :

$$C^d = M^{e.d} = M^{1+k.\phi(n)} = M^1.(M^{\phi(n)})^k$$
$$= M^1.(1)^k = M^1 = M \bmod n$$

RSA Example - Key Setup

1. Select primes: $p=17$ & $q=11$
2. Calculate $n = pq = 17 \times 11 = 187$
3. Calculate $\phi(n) = (p-1)(q-1) = 16 \times 10 = 160$
4. Select e: $\gcd(e,160)=1$; choose $e=7$
5. Determine d: $de = 1 \bmod 160$ and $d < 160$
   Value is $d=23$ since $23 \times 7 = 161 = 10 \times 160 + 1$
6. Publish public key $PU=\{7,187\}$
7. Keep secret private key $PR=\{23,187\}$

RSA Example - En/Decryption

- sample RSA encryption/decryption is:
- given message $M = 88$ (nb. $88<187$)
- encryption:

  $C = 88^7 \bmod 187 = 11$
- decryption:

  $M = 11^{23} \bmod 187 = 88$

RSA Security

- possible approaches to attacking RSA are:
  - brute force key search - infeasible given size of numbers
  - mathematical attacks - based on difficulty of computing ø(n), by factoring modulus n
  - chosen ciphertext attack (CCA), choose ciphertext to exploit properties of RSA to provide info to help cryptanalysis

Progress in Factoring

Number of Decimal Digits	Approximate Number of Bits	Date Achieved	MIPS-years	Algorithm
100	332	April 1991	7	quadratic sieve
110	365	April 1992	75	quadratic sieve
120	398	June 1993	830	quadratic sieve
129	428	April 1994	5000	quadratic sieve
130	431	April 1996	1000	generalized number field sieve
140	465	February 1999	2000	generalized number field sieve
155	512	August 1999	8000	generalized number field sieve
160	530	April 2003	—	Lattice sieve
174	576	December 2003	—	Lattice sieve
200	663	May 2005	—	Lattice sieve